DISCUSSION GUIDE

SECRETS OF THE SURFACE

The Mathematical Vision of Maryam Mirzakhani

CREATED BY IMPACT MEDIA PARTNERS LLC FOR ZALA FILMS

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USING THIS GUIDE

This discussion guide is designed to support educators in presenting the film *SECRETS OF THE SURFACE* within the context of a women and gender studies curriculum. After screening the film for their class, educators can use the discussion questions included in this guide to facilitate student reflection on the ways gender and cultural identity influenced Maryam Mirzakhani's achievements in mathematics. A variety of essay questions and a class activity focused on implicit bias encourage students to think systemically about equity in mathematics and other STEM fields. Educators will also find in this guide additional contextual information and recommended resources to supplement the information presented in *SECRETS OF THE SURFACE*.

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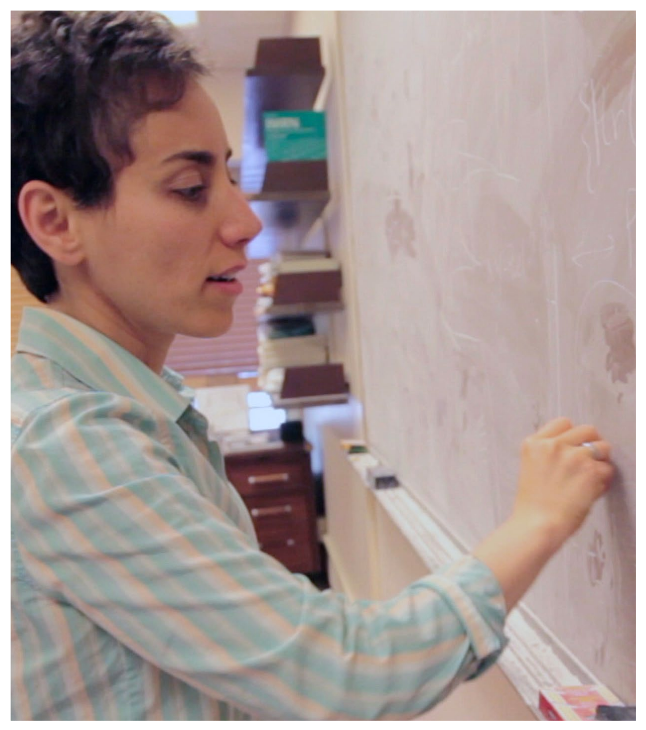


ABOUT THE FILM

Filmed in Canada, Iran, and the United States, *SECRETS OF THE SURFACE: THE MATHEMATICAL VISION OF MARYAM MIRZAKHANI* examines the life and mathematical work of Maryam Mirzakhani, an Iranian immigrant to the US who became a superstar in her field. In 2014, she was both the first woman and the first Iranian to be honored by mathematics' highest prize, the Fields Medal. Mirzakhani's contributions are explained in the film by leading mathematicians and illustrated by animated sequences. Her mathematical colleagues from around the world, as well as former teachers, classmates, and students in Iran today, convey the deep impact of her achievements. The path of her education, success on Iran's Math Olympiad team, and her brilliant work, make Mirzakhani an ideal role model for girls looking toward careers in science and mathematics.

ABOUT THE FILMMAKER

George Paul Csicsery, a writer and independent filmmaker since 1968, was born in Germany in 1948, the son of Hungarian parents. He immigrated to the US in 1951. He has directed 32 films—dramatic shorts, performance films, and documentaries—many on the subjects of mathematics and science.

Additional information and a filmography is available at **ZALAFILMS.COM**.

Questions for Thought or Discussion

Use these questions to guide a discussion after watching the film. If you are a larger number of people, consider diving into small groups of four or six to tackle select questions and report back.

01

What did you know about Maryam Mirzakhani before watching *SECRETS OF THE SURFACE*? What elements of her story stood out to you as you were watching the film?

02

What stereotypes did the film challenge? Were any stereotypes reinforced by the film? If so, which ones?

03

What were some of the early sources of support that encouraged Mirzakhani to realize her potential in mathematics?

04

How did the friendship between Mirzakhani and fellow mathematician Roya Beheshti influence their mutual success? Could this relationship have helped to mitigate some of the barriers to professional advancement that women often face?

05

Mirzakhani's teachers described her as a "child prodigy" in mathematics. How does being "exceptional" help gain access to academic opportunity?

06

Graduates from Sharif University of Technology in Tehran typically apply to study abroad for their PhDs. What effect does "brain drain" have on the advancement of women's rights within the country?

07

How did gender roles change for Mirzakhani and her peers once they left Iran? What cultural continuities did the cohort of Iranian students maintain when they studied abroad, and how do you think this contributed to Mirzakhani's success?

08

Can you name other women mathematicians in history? Why do you think women mathematicians are typically less known than their male counterparts? What factors influence recognition in official historical records?

09

Women and people of color are disproportionately underrepresented in mathematics degrees and careers in the US. What systemic factors influence the demographic composition of mathematics and other STEM fields?

Questions for Thought or Discussion (continued)

10

While she was alive, Mirzakhani rejected the publicity that came with winning the Fields Medal. Yet in her death, she has become a celebrated public figure. How is Mirzakhani's image used to promote Iran's national interests? Who or what else benefits from celebrating Mirzakhani's image as a woman mathematician?

11

What effect do you think role models like Mirzakhani have on students in male-dominated fields? How might women and girl mathematicians benefit from Mirzakhani's success?

12

What questions do you have after watching *SECRETS OF THE SURFACE*? What would you like to research more in order to help you better understand Mirzakhani's story and the themes relevant to the film?

13

Maryam wanted people to look at her mathematics, not at the fact that she was Iranian or a woman. She went out of her way to avoid politicizing her image and her role; given that, is it valid to analyze her life and work in terms of gender, ethnicity, and culture?

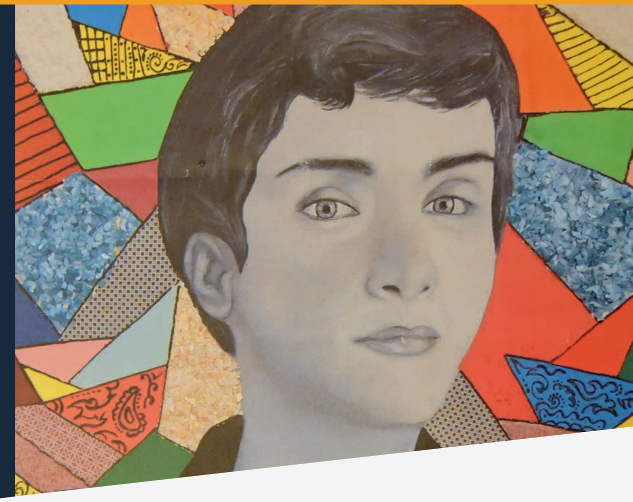
14

If Mirzakhani had grown up in the United States, how might her opportunities to succeed in mathematics have been different, either in a positive and/or negative sense?

> *"If you want to get a reasonably good idea, you have to spend a lot of time just thinking patiently, not getting disappointed somehow, coming back to the same problem. And staying somehow confident that maybe one day you will have a good idea."*
>
> – Maryam Mirzakhani

Essay Questions

Consider assigning one or more essay questions to students after watching the film to facilitate their own analysis of the issues.



01

Civil rights scholar Kimberlé Crenshaw developed the concept of intersectionality in the 1980s while analyzing the ways Black women are affected by gender and race discrimination simultaneously. Intersectionality can be used to better understand how people experience society differently depending on the intersection of their identities. How would you apply intersectionality to Maryam Mirzakhani's story? Which identities were foregrounded in the film to help us understand Mirzakhani's experiences? In what ways did intersectionality influence the opportunities available to Mirzakhani in the field of mathematics? How were Mirzakhani's experiences consistent and/or inconsistent with others who share her identities?

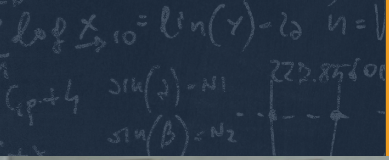
02

Reflect on your own socialization and how it has influenced your academic pursuits. Who and what influenced your early academic interests and how do you think these influences affected the trajectory of your education? How has gender socialization in particular affected your participation in mathematics education or other science fields? What experiences, comments, or subtle messages were formative in shaping your ideas about gender roles and academics?

03

Analyze the structural and systemic factors that influence equity in STEM careers in the US, Iran, or another country of interest. What does the data in your selected country indicate about the status of equity in STEM? How does the organization of the country's education system affect equity across academic disciplines? What policies or procedures in the country influence equal access to opportunity in STEM careers? How do institutions in your selected country address retention of diverse STEM professionals once they are hired? Based on your research, what conclusions can you draw about strategies for increasing equity in STEM?

**You can also use any of the discussion questions above as the basis for a more detailed essay assignment.*



In-Class or Group Activity

Depending on technology available, this activity could be done together in a classroom setting, assigned as homework, or added to an online classroom forum.

OBJECTIVE: Increase student understanding of implicit bias toward social groups and how it affects equal access to opportunities in STEM fields.

TIME: Approx. 45 minutes

MATERIALS:
To complete this activity, students will need access to:

- Computers with Internet
- LINK to Implicit Bias Test by Project Implicit at Harvard University (Approx. 5 minutes)
- LINK to video by *Christian Science Monitor* (2018) "This test reveals implicit biases you don't know you have." (Runtime: 6:20 minutes)

INSTRUCTIONS:

01. Assign students to take THE "GENDER-SCIENCE" IMPLICIT BIAS TEST. To find this test, students should follow the website prompts until arriving at a menu screen with a list of tests. Students can then select the button labeled "Gender-Science" IAT. The test takes approximately five minutes to complete.

02. After they have completed the test, students should make note of their results. Inform the students that test results do not need to be shared; they are for personal reflection only.

03. Then ask students to watch the video "This test reveals implicit biases you don't know you have" posted at this LINK.

04. Finally, facilitate a class discussion to reflect on the concept of implicit bias with your students. Sample discussion questions include:

    a. Based on the test and video in this class activity, how would you define implicit bias in your own words?

    b. What was your experience taking the test? What was going through your head as you were completing it?

    c. What is your assessment of the test design? Do you think it can accurately detect implicit bias? Why or why not?

    d. Based on the video you watched, how might implicit bias affect equal opportunity?

    e. What strategies did the video recommend to address implicit bias? What are the strengths and weaknesses of the recommendations suggested?

Contextual Information

Additional context can be used by educators to inform classroom discussion or be provided directly to students as supplemental reading before or after watching the film.

About Maryam Mirzakhani

Maryam Mirzakhani gained worldwide acclaim when she won the coveted Fields Medal for her work in hyperbolic geometry. Considered the Nobel Prize of mathematics, the Fields Medal is awarded every four years to outstanding mathematicians 40 years old or younger. At age 37, Mirzakhani was the first woman and first Iranian to receive the award since its founding in 1936.

Born in 1977 in Iran, Mirzakhani spent her early years as an avid reader and dreamed of becoming a writer. After the Iranian Revolution, she attended the Farzanegan school for gifted girls in Tehran. She discovered her aptitude for mathematics through the support of her family and the camaraderie of her grade school classmate and fellow mathematician Roya Beheshti, who is featured in the film. In 1994 as high school juniors, Mirzakhani and Beheshti were the first young women to compete on the Iranian national team in the International Mathematical Olympiad. Mirzakhani earned gold medals both years she competed—the second year with a perfect score.

Mirzakhani studied mathematics at Sharif University of Technology in Tehran and then moved to the US in 1999 to pursue her doctorate at Harvard University. Mirzakhani was a 2004 research fellow of the Clay Mathematics Institute and a professor at Princeton University. In 2008, she became a professor at Stanford University. She married fellow mathematician, Jan Vondrák, and had one daughter.

Throughout her academic career, she solved a number of problems related to the geometry of moduli spaces—abstract shapes of monumental proportions in which every point represents a surface. Together with mathematicians Alex Eskin and Amir Mohammadi, she discovered what is known as the "magic wand" theorem used to understand a classic physics problem about the motion of a billiard ball on a polygonal table.

When awarded the Fields Medal in Seoul in 2014, she was in the midst of a battle with breast cancer, which ultimately took her life in July of 2017. Although during her life she avoided the spotlight, today she is considered a hero in Iran and by mathematicians worldwide. Mirzakhani's birthday on May 12th is celebrated by women in mathematics associations around the world.

Sources:

Lamb, Evelyn (2017) "Mathematics World Mourns Maryam Mirzakhani, Only Woman to Win Fields Medal," *Scientific American*.

Cook, Garett (2017) "The Lives They Lived: Maryam Mirzakhani," *The New York Times Magazine.*

The Mathematics of Maryam Mirzakhani

By Erica Klarreich

A lot of Mirzakhani's work had to do with the geometry of surfaces, like the surface of a ball, or the surface of a doughnut, what mathematicians call a torus. And in her doctoral dissertation, she focused on a very basic question about surface geometry, which is about what happens if you start somewhere on the surface and walk in a straight line. Depending on your surface, your path might close up on itself. For example, if you're walking straight on the surface of the Earth, you're going to go around a great circle like the equator and end up back where you started.

Mirzakhani's doctoral dissertation focused on the straight paths that eventually close up, and never cut across themselves—what mathematicians call "simple closed curves." These curves are easy to categorize on a surface like a sphere or a doughnut (what mathematicians call a torus), but not easy to categorize on a torus with more than one hole.

To understand the simple closed curves on a surface, you have to know the surface's precise geometry. And when we talk about a "torus" or a "torus with two holes," we've said something about the shape of the surface, but not everything. We could mean a really plump torus, or maybe a long, skinny torus.

Mathematicians have a concept called the "moduli space" that lets you consider all the different possible geometries on a surface in one fell swoop. This moduli space is like a map, and every point on the map is the address for a different possible geometry that could live on your surface. By looking at the moduli space, Mirzakhani was able to figure out how to count the simple closed curves of different lengths on lots of different kinds of surfaces.

Another problem that she solved stems from a very simple question: If you hit a ball on a billiard table, what are the possible ways that it could travel? Maybe it's going to roll into a corner, or bounce around in a cycle, or follow a more complicated trajectory. And here we might mean the usual rectangular billiard table, or we might take a more complicated shape like a triangle or a hexagon. For most of these different shaped billiard tables, it's really hard to understand what the different possible trajectories are.

But there's a way to transform this problem into a problem about the geometry of surfaces, which was one of Mirzakhani's specialties. Imagine that we put a mirror along every side of our billiard table. When a ball hits one of the walls, it's going to bounce off, but in the mirror, it looks as if it just rolled in a straight line into a looking-glass world. And when it hits the next wall, it will roll straight into another looking-glass world.

For the kind of billiard tables Mirzakhani studied, it turns out as the ball rolls through this series of looking-glass worlds, it will eventually get to one that's a perfect copy of your original billiard table, just shifted over. Any further rolling is just going to give you repeats of things you've already seen. We can imagine that we take the last table before the repeats started and glue it to our original table, instead of unfolding it onto a new table. And if you do all these different gluings, you end up with a surface. And this surface gets a very precise geometry that comes from the billiard table.

This geometry is not the only billiard table geometry we could imagine our surface having. Maybe there's some other billiard table that glues up into the same surface, but with a different geometry that maybe makes the handles fatter or skinnier. Just as with Mirzakhani's earlier work, we can talk about a moduli space of different geometries.

To understand specific billiard tables, Mirzakhani used an approach that looks at all the different billiard geometries at the same time, in the moduli space. Studying this moduli space enabled her to figure out things like how many ways a billiard ball can bounce back to itself, or how a billiard ball spreads around on a table. The theorem she and her collaborator Alex Eskin proved about billiard table geometries was so powerful that mathematicians call it the Magic Wand theorem.

Gender Equity in Iranian Education

In 1979, two years after Mirzakhani was born, the Iranian Revolution swept through the country and a new Islamic Republic was formed under the leadership of Ayatollah Ruhollah Khomeini. At the time, university enrollment for women was at a high, although often inaccessible to women in more rural, religiously conservative areas. Under Ayatollah Khomeini, Iranian schools were reformed and reopened with sex segregation and restrictions for female students' behavior and dress code. More girls from conservative religious families enrolled in school and the gender gap in the education system closed rapidly over the next few decades. (Winn, 2016)

Women's enrollment in university has also steadily increased, surpassing men's enrollment in many degrees. During the ten years between 1993 to 2003, women represented 68 percent of bachelor's degree, 52 percent of master's degree and 35 percent of all doctorate degree entrants of all academic disciplines in Iranian universities. (Rahbari, 2016) In response to this trend, the Iranian parliament issued gender quotas in public universities arguing that investing in educating women in particular fields did not make sense given the limited career paths for them. As part of the quota, 36 universities redefined 77 fields of study as "single gendered." (Tait, 2012) Many science fields were among those restricted to male only, including 20 percent of mathematics majors, while several degrees such as nursing were designated as female only. Mirzakhani's alma mater Sharif University of Technology was not among the list of universities that implemented the new restrictions.

Despite the high number of female students enrolled in Iranian universities, the numbers of women professors in higher education and in the workforce are both small. In 2009, women occupied 18 percent of university tenure track positions in Iran and, in the larger workforce, a similar percentage of women worked outside the home. (Rahbari, 2016) A struggling economy is partly to blame, but women's rights advocates also criticize gender discrimination in hiring and conservative patriarchal values that limit women's participation in the public sphere.

Sources:

Winn, Meredith Katherine (2016) "Women in Higher Education in Iran: How the Islamic Revolution Contributed to an Increase in Female Enrollment," Global Tides: Vol. 10, Article 10.

Tait, Robert (2012) "Anger as Iran bans women from universities." The Telegraph

Rahbari, Ladan (2016) "Women in Higher Education and Academia in Iran," *Sociology and Anthropology* 4(11): 1003-1010.

Iranian International Students

Sharif University of Technology is a premiere academic institution in Iran and the world, and its graduates are frequently accepted for student visas abroad to continue their graduate studies. There are as many as 50,000 Iranians studying around the world, including 8,700 studying in the US, according to a 2014 study by the Washington Institute for Near East Policy. (Krever, 2017) The large number of students that leave Iran to study abroad raises concerns about "brain drain," a term used to describe the effects of highly talented and trained Iranians emigrating from the country as Mirzakhani did.

Gaining a student visa to study in the United States requires significant preparation and expense. It is even more difficult for Iranian students. Because there is no US embassy in Iran, students must travel abroad to complete the interviews, in addition to submitting extensive documentation detailing their backgrounds, education, work experience, family ties, as well as social media accounts and email addresses.

Even after obtaining a student visa, admittance into the country has become vulnerable to the shifting political relations between the US and Iran. Under President Trump's administration, Iranian students have been turned away by US Customs and Border Protection that deemed them "inadmissible" despite being previously vetted by the US State Department in the visa process. After being sent back to Iran, many students also received a five-year ban from entering the US.

Iran is one of 13 countries included in the travel ban issued by President Trump; however the ban exempts student visas for Iranians. In the 2018-2019 academic year while the ban was in effect, 1800 Iranian students were issued visas. (Abdalla, 2020)

Sources:

Krever, Mick (2017) "Brain drain to the West," CNN.

Abdalla, Jihan (2020) "Iranian students with valid visas turned back at US borders," Al Jazeera.

Gender and Race Equity in STEM in the US

Like in Iran, women and girls are disproportionately underrepresented in the fields of science, technology, engineering, and mathematics (STEM) in countries worldwide. This is especially true for women and girls from racial and ethnic minority groups.

In the US, for example, female students outnumber male students in every degree level in academia. However in mathematics and statistics during the 2015-2016 academic year, women earned 43 percent of Bachelor's degrees, 42 percent of Master's degrees, and just 29 percent of PhDs. (Catalyst, 2019) And while women took home approximately one third of all STEM degrees in the US, women of color accounted for a small fraction of this total; Asian women earned five percent, Black women three percent, and Latinas four percent of STEM degrees in 2015-2016. (Ibid)

Advocates argue that having a greater diversity among role models and educators in STEM fields is an important component in achieving parity in STEM education. Yet, here the numbers also fall short. In 2016, women represented just 15 percent of tenure track positions in mathematics—one of the lowest percentages among the sciences alongside computer science (18 percent) and engineering (14 percent). (Hu, 2016)

In the overall US workforce, women made up less than one-quarter of those employed in STEM professions in 2015, and fewer than two in ten science and engineering employees were women of color. (Catalyst, 2019) While outright employment discrimination is prohibited in the US, advocates have identified a number of barriers to equity in STEM professions, including implicit bias by hiring committees, unwelcoming work environments, and unsupportive employment policies around parental leave. In effect, these barriers exclude many women and people of color from lucrative STEM careers, exacerbating economic disparities among social groups.

Sources:

Catalyst (2019) Quick Take: Women in Science, Technology, Engineering, and Mathematics (STEM).

Hu, Jane C. (2016) "Why Are There So Few Women Mathematicians?" The Atlantic.

Women in Mathematics History

Mirzakhani was the first woman to be recognized with a Fields Medal, but she was certainly not the first woman to make significant contributions to the field of mathematics. Here are a few of the notable women in mathematics history:

- **Hypatia** (b. 355 - 415) - Greek mathematician, astronomer, philosopher, and one of the first documented scholars.
- **Sutayta Al-Mahamali** (d. 987) - Algebraist and legal scholar who lived in Baghdad, and solved problems of inheritance in Islamic law using Algebra.
- **Sophie Germain** (1776 - 1831) - French mathematician who assumed a man's identity to solve

problems in acoustics, elasticity, and the theory of numbers.

- **Ada Lovelace** (1815 - 1852) - English aristocrat who wrote the world's first computer program to calculate a sequence of Bernoulli numbers.
- **Sofia Vasilyevna** (1850 - 1891) - Russian professor of mathematics at the University of Stockholm whose essay on the rotation of a solid body earned her several prestigious awards.
- **Emmy Noether** (1882 - 1935) - A Jewish mathematician who grew up in Germany before fleeing to the US where she developed many of the mathematical foundations for Einstein's general theory of relativity.
- **Katherine Johnson** (1918 - 2020) - An African-American mathematician at NASA who calculated the orbital mechanics for the first US space flight.
- **Julia Hall Bowman Robinson** (1919 – 1985) - An American mathematician noted for her contributions to the fields of computability theory and computational complexity theory—most notably in decision problems.

Roots of Mathematics from the Islamic Golden Age

Mirzakhani's achievements continue a legacy of scholarship from the Islamic world in the field of mathematics that dates back to the 9th century. During that time, Islam and the empire that embraced it were expanding rapidly, covering territories from Spain to the borders of China. The conquest of lands often allowed scholars to acquire foreign intellectual works. Academic centers, such as the House of Wisdom in Baghdad, oversaw the translation of many Greek and Hindu works on mathematics into Arabic—including the 10-digit number system first developed by Hindu scholars and used throughout the world today.

Building on these translated texts, scientists from the Islamic world—whether they themselves were Muslim or not—led three major mathematical advancements in the 10th century: the completion of arithmetic algorithms, the development of algebra, and the extension of geometry. In the 11th century, Omar Khayyam (who hailed from what today is known as Iran and is perhaps more often known for his poetry) was instrumental in continuing many of the projects of 10th century scholars, including solving problems of roots and cubic equations. The scholastic inheritance of mathematicians from the time of the Islamic Golden Age is visible throughout the Middle East in the elaborate geometric architecture of the world's oldest mosques.

"She's (Maryam has) opened up a new way. She has proved that it doesn't matter being the first one. You can do anything that you want to when you put your desire and your effort in."

- Dorsa Majdi,
a student at the Farzanegan School for Girls in Tehran

Recommended Reading

These resources can be assigned to students as required or recommended reading to inform their engagement with the film.

Carland, Susan (2017) *Fighting Hislam: Women, Faith and Sexism*, Melbourne University Press.

Crenshaw, Kimberlé (1989) "Demarginalizing the Intersection of Race and Sex: A Black Feminist Critique of Antidiscrimination Doctrine, Feminist Theory and Antiracist Politics," University of Chicago Legal Forum: Vol 1989, Issue 1, Article 8
Available at: CHICAGOUNBOUND.UCHICAGO.EDU/UCLF/VOL1989/ISS1/8

Ross V., Guerrero S., Fenton E. (2016) Mathematics Teaching and Learning: Equality ≠ Equity. In: Papa R., Eadens D., Eadens D. (eds) Social Justice Instruction. Springer, Cham
Available at: LINK.SPRINGER.COM/CHAPTER/10.1007/978-3-319-12349-3_17

Winn, Meredith Katherine (2016) "Women in Higher Education in Iran: How the Islamic Revolution Contributed to an Increase in Female Enrollment," Global Tides: Vol. 10 , Article 10.
Available at: DIGITALCOMMONS.PEPPERDINE.EDU/GLOBALTIDES/VOL10/ISS1/10/

Yang, Yang; Carroll, Doris Wright (2018) "Gendered Microaggressions in Science, Technology, Engineering, and Mathematics," *Leadership and Research in Education*, Vol. 4 spec issue p28-45, 2017-2018.
Available at: ERIC.ED.GOV/?ID=EJ1174441